

exhibit A
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repr. → random diagrams
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shape ↔ character
oooooooooooo

exhibit B
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RSK
oooo

bumping routes
oooooo

S_∞
ooooo

the end
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museum of visual ART

Asymptotic Representation Theory

guided tour with Piotr Śniady

transparencies, references, homework available on
<http://psniady.impan.pl/fpsac>

exhibit A
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one-slide summary

randomness

combinatorics



← representation theory

$$V^\lambda \downarrow_{S_m}^{S_n}$$

visual viewpoint on algebraic combinatorics creates nice questions

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one-slide summary

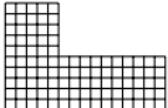
randomness

asymptotic $n \rightarrow \infty$

representation theory

$$V^\lambda \downarrow_{S_m}^{S_n}$$

combinatorics



visual viewpoint on algebraic combinatorics creates nice questions

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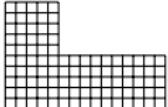
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visual

visual viewpoint on algebraic combinatorics creates nice questions

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plan for today

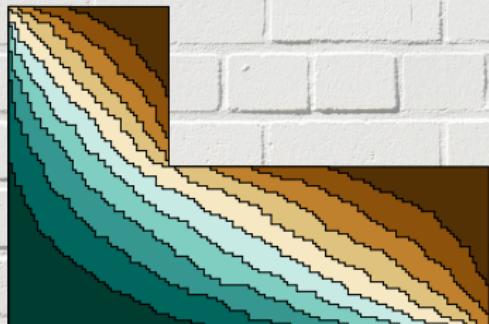


exhibit A

what can you say
about random Young diagrams?

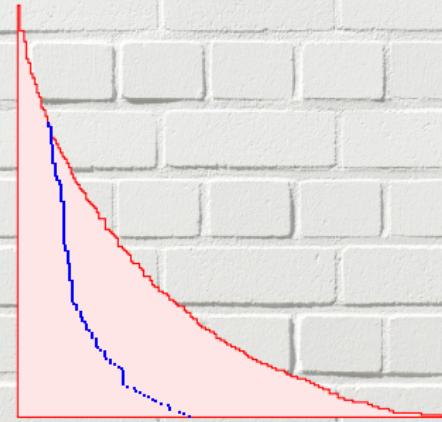


exhibit B

what can you say
about RSK
applied to random input?

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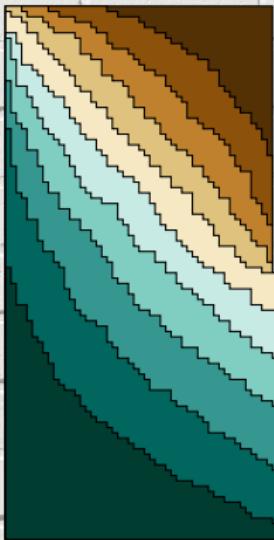
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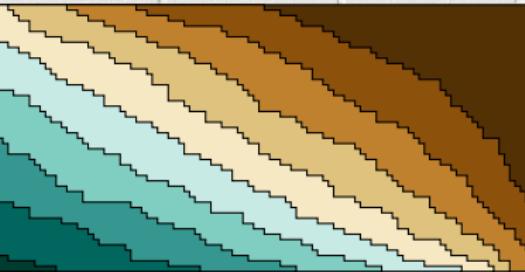
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exhibit A



how this picture was created?

- 1 2 3 4



what can we say

about random Young diagrams and random Young tableaux?

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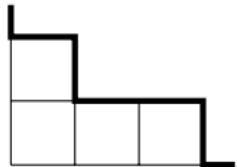
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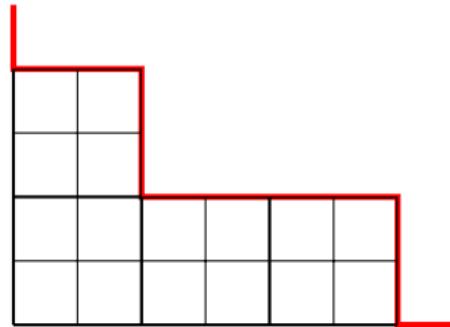
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1 scaling



ξ

start with a Young
diagram ξ ...



$\lambda = s\xi$

... and scale it
by a factor $s \in \{1, 2, \dots\}$

if $s > 0$ is a real number, $s\xi$ is a *generalized Young diagram*

$\Lambda = \frac{1}{\sqrt{|\lambda|}}\lambda$ is called *asymptotic shape of λ*

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② select randomly a standard tableau with shape λ

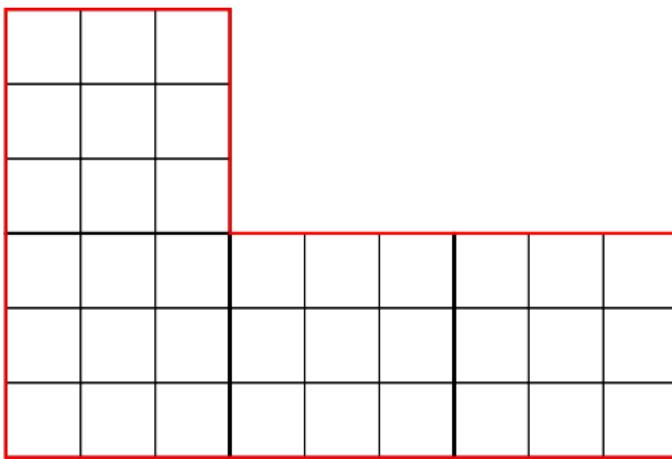


diagram λ

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② select randomly a standard tableau with shape λ

20	23	31							
9	15	30							
8	14	28							
6	7	12	17	22	26	29	33	36	
3	4	11	13	18	21	25	32	35	
1	2	5	10	16	19	24	27	34	

tableau T

(3) draw the level curves

fix a real number $0 \leq \alpha \leq 1$

$$n = |\lambda|$$

$$m = \lfloor \alpha n \rfloor$$

 $\mu = T_{\leq \alpha n} = (\text{boxes of } T \text{ which are } \leq \alpha n)$ is a random Young diagram with m boxes

20	23	31
9	15	30
8	14	28
6	7	12
17	22	26
29	33	36
3	4	11
13	18	21
25	32	35
1	2	5
10	16	19
24	27	34

level curve $\alpha = \frac{1}{2}$

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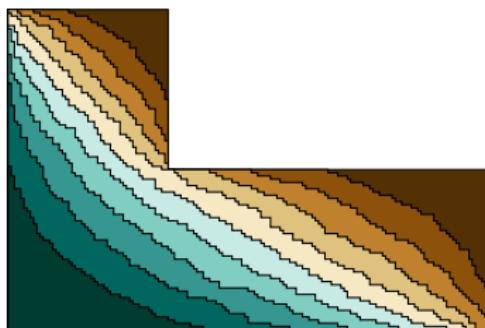
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④ this is a layer tinting of a random standard tableau of fixed shape λ



Population, Landscape and Climate Estimates, v3:
Elevation Zones, South America

National Aggregates of Geospatial Data Collection

PLACE III



hint: $\lambda = s\xi$

for best viewing experience
scale the picture by $\frac{1}{s}$

then $s \rightarrow \infty$

Digital elevation data were obtained as a 3-km resolution elevation/bathymetry raster product from iScience, LLC. Elevation zones were created by aggregating ranges of land elevation values and sea surface bathymetry values. This product combines NASA's Shuttle Radar Topographic Mission (SRTM30) digital elevation data with bathymetry values to produce a seamless, globally consistent land elevation and marine depth layer. Gap and void areas were filled using a combination of elevation data layers from the NOAA GLOBE project to provide a high-quality global coverage of all land surface areas.

Credit for International Earth Science Information Network (IESIN) and the National Geospatial Data Committee (NGDC) is given to the University of California in the City of New York, Center for Interpreting Earth Science Information Network (CIESIN) (Columbia University, 2012). National Aggregates of Geospatial Data Collection: Population, Landscape and Climate Estimates, v3.0, PLACE III, National Geospatial Data Committee, Columbia University, Center for International Earth Science Information Network (CIESIN), University of California in the City of New York, Center for Interpreting Earth Science Information Network (CIESIN) (Columbia University, 2012).

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March 2012

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reducible representation \longrightarrow random Young diagram μ

let W be a reducible representation of the symmetric group S_m ;
 its decomposition into irreducibles is given by

$$W = \bigoplus_{\mu \vdash m} m_\mu V^\mu$$

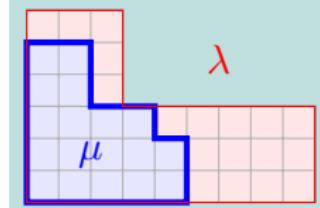
$m_\mu \in \{0, 1, \dots\}$ is the multiplicity of V^μ in W

we declare the probability of sampling the Young diagram μ to be equal to

$$\mathbb{P}_W(\mu) = \frac{m_\mu \dim V^\mu}{\dim W}$$

our concrete example

$$W = V^\lambda \downarrow_{S_m}^{S_n}$$



"random irreducible component of W "

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the end
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framework

cycles

$$\begin{aligned}[k] &= (1, \dots, k) && \in S_m && \text{for } m \geq k, \\ [k, \ell] &= (1, \dots, k)(k+1, \dots, k+\ell) && \in S_m && \text{for } m \geq k+\ell\end{aligned}$$

characters

$$\chi_W(\pi) = \frac{\text{Tr} \rho_W(\pi)}{\text{Tr} \rho_W(\text{id})} \quad \text{for } \pi \in S_m$$

asymptotic setting

somebody gives us some interesting sequence W_1, W_2, \dots

W_m is a (reducible or irreducible) representation of S_m ,

$\chi_m = \chi_{W_m}$ is its character

$\lambda^{(m)}$ is a random Young diagram with m boxes,
the random irreducible component of W_m

exhibit A

repr. → random diagrams
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shape \leftrightarrow character
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RSK
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bumping routes oooooo

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limit shape \longleftrightarrow characters

the following two conditions are equivalent
(if you add sufficiently many technical assumptions):

the asymptotic shape $\frac{1}{\sqrt{m}}\lambda^{(m)}$ converges to some limit shape Λ

the character χ_m has a specific asymptotic behavior which depends on the limit shape Λ

→ Philippe Biane 1998, 2001

exhibit A
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shape ↔ character
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limit shape → characters

start with a Young diagram ξ

$$\lambda^{(n)} := s\xi \quad \text{if } n \text{ is of the form } n = s^2|\xi|,$$

$$\text{in this way } \frac{1}{\sqrt{n}}\lambda^{(n)} = \frac{1}{\sqrt{|\xi|}}\xi = \Lambda,$$

$$W_n = V^{\lambda^{(n)}}$$

Theorem (Philippe Biane 1998)

for each $k \in \{1, 2, \dots\}$ the limit

$$R_{k+1}(\Lambda) := \lim_{n \rightarrow \infty} \chi_n([k]) n^{\frac{k-1}{2}}$$

exists and is a nice function of the limit shape Λ

R_2, R_3, \dots are called **free cumulants** of Λ

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characters → limit shape

- assume that for each $k \in \{1, 2, \dots\}$ the limit exists

$$R_{k+1} := \lim_{m \rightarrow \infty} \chi_m([k]) m^{\frac{k-1}{2}}$$

- assume that

$$\chi_m([k, l]) \approx \chi_m([k]) \chi_m([l]) \quad \text{for } m \rightarrow \infty;$$

let a random Young diagram $\mu^{(m)}$
 be a random irreducible component of W_m

Theorem (Philippe Biane 2001)

$\frac{1}{\sqrt{m}} \mu^{(m)} \xrightarrow[m \rightarrow \infty]{\text{in probability}} \text{generalized Young diagram}$

with free cumulants R_2, R_3, \dots

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exhibit A: why the limit curves exist?

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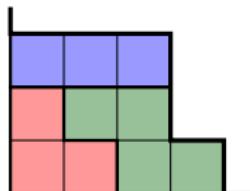
why shape of $\lambda \rightarrow$ character of V^λ ?

classic tools:

Murnaghan–Nakayama rule

$$\pi = [3, 4, 3]$$

$$\text{Tr } \rho^\lambda(\pi) = (-1)^0 \cdot (-1)^1 \cdot (-1)^1 + \dots$$



lesson: old combinatorics is not useful today

can algebraic combinatorics
provide new exact formulas for characters
which are useful for asymptotic questions?

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dual viewpoint on characters

for a Young diagram λ with n boxes
and $k \in \{1, 2, \dots\}$ we define

$$\text{Ch}_k(\lambda) = \begin{cases} \underbrace{n \cdot (n-1) \cdots (n-k+1)}_{k \text{ factors}} \chi_\lambda([k]) & \text{if } n \geq k, \\ 0 & \text{if } n < k, \end{cases}$$

→ Ivanov, Kerov 1999

for each integer $k \geq 1$ and each Young diagram λ

$$\{1, 2, \dots\} \ni s \mapsto \text{Ch}_k(s\lambda)$$

is a polynomial of degree $k + 1$

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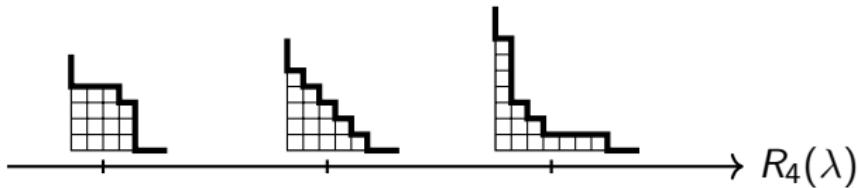
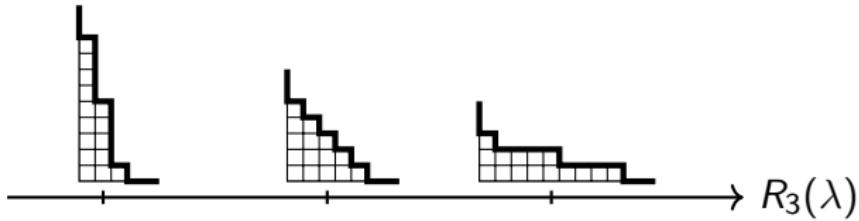
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free cumulants \longleftrightarrow shape

$$R_{k+1}(\lambda) = [s^{k+1}] \text{Ch}_k(s\lambda) = \lim_{s \rightarrow \infty} \frac{1}{s^{k+1}} \text{Ch}_k(s\lambda)$$



“asymptotic shape=free cumulants”

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Kerov positivity conjecture

Biane's results are based on

$$\text{Ch}_k \approx R_{k+1}$$

$$\overbrace{\text{Ch}_2}^{\text{character}} = \overbrace{R_3}^{\text{shape}},$$

$$\text{Ch}_3 = R_4 + R_2,$$

$$\text{Ch}_4 = R_5 + 5R_3,$$

$$\text{Ch}_5 = R_6 + 15R_4 + 5R_2^2 + 8R_2,$$

$$\text{Ch}_6 = R_7 + 35R_5 + 35R_3R_2 + 84R_3$$

why positivity?

→ Stanley–Féray character formula

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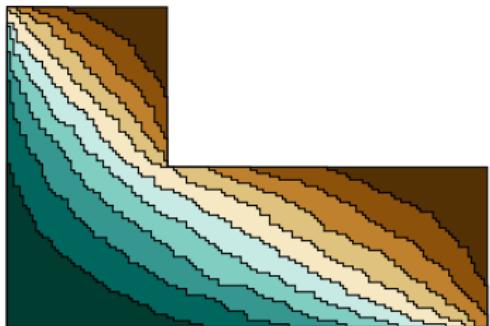
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exhibit A: moral lessons



- Biane's machinery has many more applications!
homework → <http://psniady.impan.pl/fpsac>

- some classical tools of algebraic combinatorics are not convenient for asymptotic questions,
- asymptotic viewpoint may create new ("dual") tools in algebraic combinatorics,
- without asymptotic motivations you would not look for new character formulas,

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outlook: Lassalle's conjecture

characters of the symmetric groups $\text{Ch}_n =$
dual viewpoint on **Schur polynomials**

Jack characters $\text{Ch}_k^{(\gamma)} =$ dual viewpoint on **Jack polynomials**
(toy example of **Macdonald polynomials**)

$$\text{Ch}_1^{(\gamma)} = R_2,$$

$$\text{Ch}_2^{(\gamma)} = R_3 + \gamma R_2,$$

$$\text{Ch}_3^{(\gamma)} = R_4 + 3\gamma R_3 + (1 + 2\gamma^2)R_2,$$

$$\text{Ch}_4^{(\gamma)} = R_5 + 6\gamma R_4 + \gamma R_2^2 + (5 + 11\gamma^2)R_3 + (7\gamma + 6\gamma^3)R_2,$$

why positivity?

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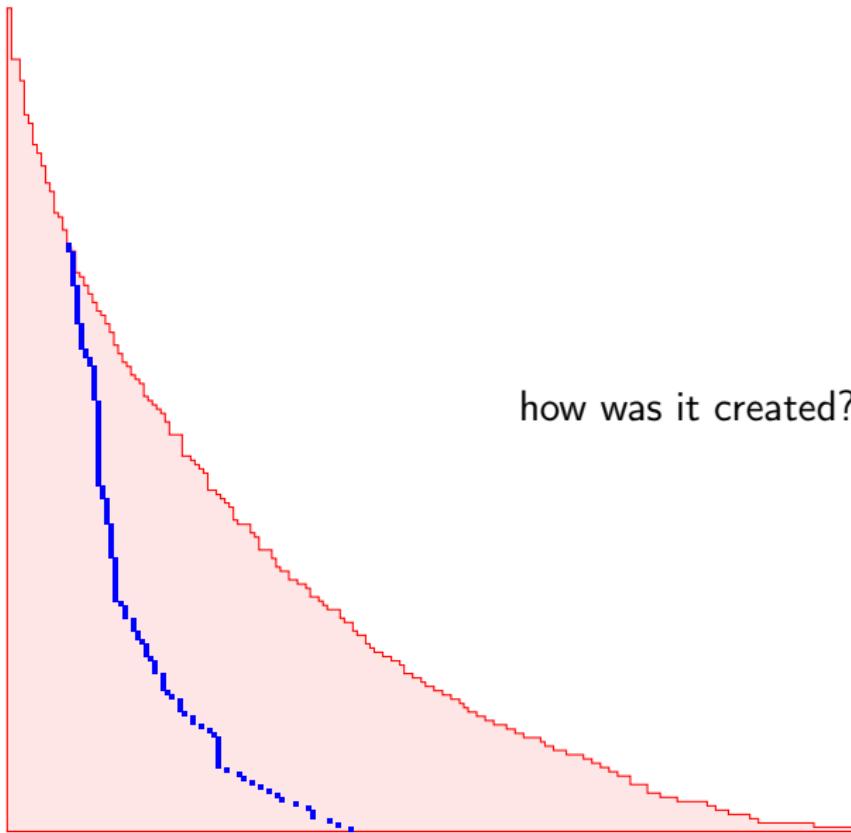
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exhibit B



how was it created? → RSK

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Robinson–Schensted–Knuth algorithm is a bijection...

output:

input:

- word $w = (w_1, \dots, w_n)$

- semistandard tableau P ,
- standard tableau Q ,

tableaux P and Q have the same shape with n boxes

example:

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41)$$

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

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Robinson-Schensted-Knuth algorithm — induction step

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recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
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Robinson-Schensted-Knuth algorithm — induction step

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Robinson-Schensted-Knuth algorithm — induction step

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18 ↗

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74	99					
23	53	70				
16	37	41	82			

insertion tableau $P(w)$

8	9					
4	6	7				
1	2	3	5			

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

insertion tableau $P(w)$

74	99		
23	53	70	
16	41	82	

37
18

recording tableau $Q(w)$

8	9		
4	6	7	
1	2	3	5

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

37 ↗

74	99			
23	53	70		
16	18	41	82	

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

37 ↗

74	99			
23	53	70		
16	18	41	82	

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

37 ↗

74	99			
23	53	70		
16	18	41	82	

insertion tableau $P(w)$

8	9			
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

insertion tableau $P(w)$

74	99			
23	53	70		
16	18	41	82	
37				

recording tableau $Q(w)$

8	9		
4	6	7	
1	2	3	5

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

insertion tableau $P(w)$

74	99			
23		70		
16	18	41	82	

Arrows indicate the path of the number 18 from the input word w to its position in the tableau. The path starts at index 18 (the value 18) and moves up to index 23 (the value 23), then right to index 37 (the value 70), then up to index 53 (the value 99), and finally right to its final position at index 18 (the value 18).

recording tableau $Q(w)$

8	9		
4	6	7	
1	2	3	5

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A

repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

exhibit B

RSK
○●○○

bumping routes
oooooo

S_∞

the end
oo

Robinson-Schensted-Knuth algorithm — induction step

53	(74	99
23	37	70	
16	18	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A

repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

exhibit B

RSK

bumping routes
oooooo

5

the end
oo

Robinson-Schensted-Knuth algorithm — induction step

53	(74	99
23	37	70	
16	18	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

w = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

74	99		
23	37	70	
16	18	41	82

53 ↗

8	9		
4	6	7	
1	2	3	5

insertion tableau $P(w)$ recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

insertion tableau $P(w)$

74	99			
53	74	99		
	23	37	70	
	16	18	41	82

recording tableau $Q(w)$

8	9		
4	6	7	
1	2	3	5

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

23	37	70	
16	18	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
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RSK
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bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm — induction step

74 (↗)

53	99					
23	37	70				
16	18	41	82			

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○●○○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm — induction step

The diagram shows the insertion tableau $P(w)$ with a red horizontal bar above it. The tableau consists of four rows:

53	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

The diagram shows the recording tableau $Q(w)$ with a red horizontal bar above it. The tableau consists of five rows:

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○●○○bumping routes
○○○○○ S_∞
○○○○○the end
○○

Robinson-Schensted-Knuth algorithm — induction step

53	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○●○○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm — induction step

74				
53	99			
23	37	70		
16	18	41	82	

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○●○○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm — induction step

74				
53	99			
23	37	70		
16	18	41	82	

insertion tableau $P(w)$

8	9			
4	6	7		
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○●○○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm — induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

10			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○●○○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm — induction step

bumping route

74			
53	99		
23	37	70	
16	18	41	82

new box

10			
8	9		
4	6	7	
1	2	3	5

insertion tableau $P(w)$

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, \textcolor{blue}{18})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○●○○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm — induction step

74			
53	99		
23	37	70	
16	18	41	82

insertion tableau $P(w)$

10			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82, 37, 41, 18)$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

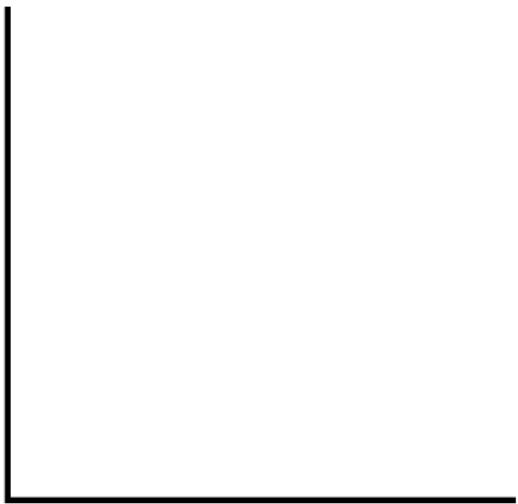
RSK
○○●○

bumping routes
○○○○○

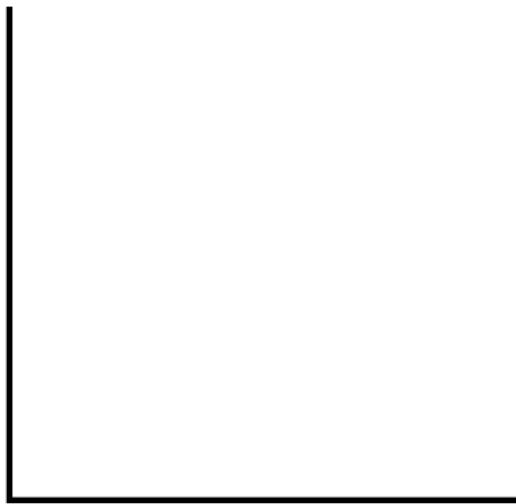
S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm



insertion tableau $P(w)$



recording tableau $Q(w)$

$$w = \emptyset$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○○●○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm

23

insertion tableau $P(w)$

1

recording tableau $Q(w)$

$w = (23)$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○○●○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm

23	53

insertion tableau $P(w)$

1	2

recording tableau $Q(w)$

$$w = (23, \textcolor{blue}{53})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
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RSK
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bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm

23	53	74

insertion tableau $P(w)$

1	2	3

recording tableau $Q(w)$

$$w = (23, 53, \textcolor{blue}{74})$$

exhibit A
ooooo

repr. → random diagrams
○

shape \leftarrow character
oooooooooooo

exhibit B

RSK
oo●o

bumping routes

5

the end
oo

Robinson-Schensted-Knuth algorithm

Category	Frequency
16	23
53	53
74	74

insertion tableau $P(w)$

4	1	2

recording tableau $Q(w)$

$w = (23, 53, 74, 16)$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
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bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm

23				
16	53	74	99	

insertion tableau $P(w)$

4				
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{red}{99})$$

exhibit A
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repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○○●○

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm

23	74
16	53
70	99

insertion tableau $P(w)$

4	6
1	2
3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{green}{99}, \textcolor{blue}{70})$$

exhibit A
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repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
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bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm

23	74	99	
16	53	70	82

insertion tableau $P(w)$

4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, 99, 70, 82)$$

exhibit A

repr. → random diagrams
○

shape \leftarrow character
oooooooooooo

exhibit B

RSK
oo●o

bumping routes oooooo

S_∞

the end
oo

Robinson-Schensted-Knuth algorithm

74			
23	53	99	
16	37	70	82

insertion tableau $P(w)$

8			
4	6	7	
1	2	3	5

recording tableau $Q(w)$

w = (23, 53, 74, 16, 99, 70, 82, 37)

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
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bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm

74	99		
23	53	70	
16	37	41	82

insertion tableau $P(w)$

8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{green}{99}, 70, 82, 37, \textcolor{blue}{41})$$

exhibit A

repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

exhibit B

RSK
○○●○

bumping routes
oooooo

5

the end
oo

Robinson-Schensted-Knuth algorithm

74			
53	99		
23	37	70	
16	34	41	82

insertion tableau $P(w)$

10			
8	9		
4	6	7	
1	2	3	5

recording tableau $Q(w)$

w = (23, 53, 74, 16, 99, 70, 82, 37, 41, 34)

exhibit A
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repr. → random diagrams
○

shape ↔ character
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exhibit B
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RSK
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bumping routes
○○○○○

S_∞
○○○○○

the end
○○

Robinson-Schensted-Knuth algorithm

74				
53	99			
23	37	70	82	
16	34	41	73	

insertion tableau $P(w)$

10				
8	9			
4	6	7	11	
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{green}{99}, 70, 82, 37, 41, 34, \textcolor{blue}{73})$$

exhibit A
00000

repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

exhibit B

RSK
○○●○

bumping routes oooooo

S_∞ the end
oooooo oo

Robinson-Schensted-Knuth algorithm

	74			
	53			
	23	99		
	16	37	70	82
	2	34	41	73

insertion tableau $P(w)$

12				
10				
8	9			
4	6	7	11	
1	2	3	5	

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{blue}{99}, 70, 82, 37, 41, 34, 73, \textcolor{red}{2})$$

exhibit A
00000

repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

exhibit B

RSK
○○●○

bumping routes oooooo

the end

Robinson-Schensted-Knuth algorithm

	74		
53	99		
23	37		
16	34	70	82
2	24	41	73

insertion tableau $P(w)$

12			
10	13		
8	9		
4	6	7	11
1	2	3	5

recording tableau $Q(w)$

$$w = (23, 53, 74, 16, \textcolor{blue}{99}, 70, 82, 37, 41, 34, 73, 2, \textcolor{red}{24})$$

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
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exhibit B
○

RSK
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bumping routes
○○○○○

S_∞
○○○○○

the end
○○

magic symmetries of RSK

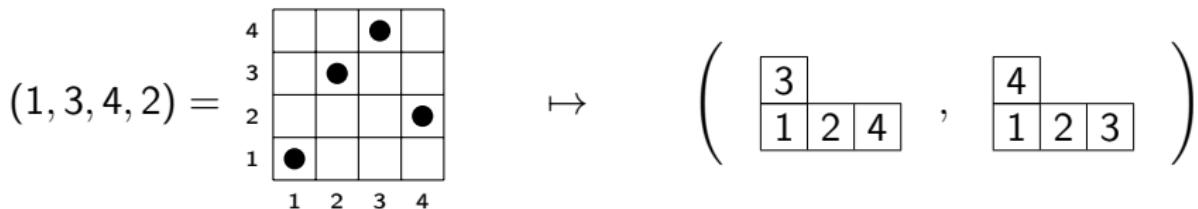


exhibit A
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repr. → random diagrams
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shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○○○●

bumping routes
○○○○○

S_∞
○○○○○

the end
○○

magic symmetries of RSK

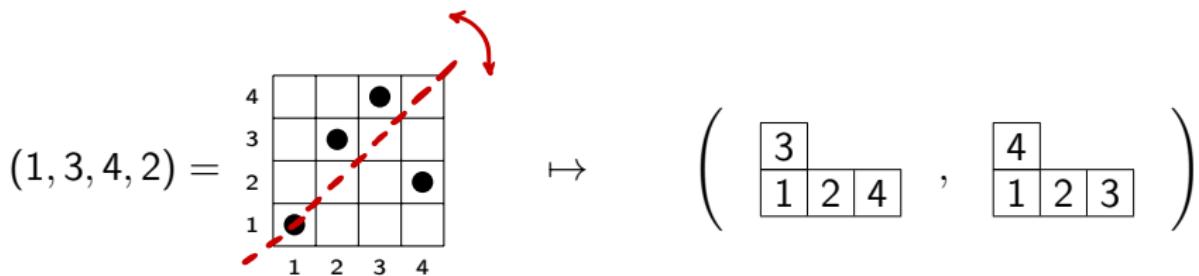


exhibit A
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repr. → random diagrams
○

shape ↔ character
○○○○○○○○○○○○

exhibit B
○

RSK
○○○●

bumping routes
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S_∞
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the end
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magic symmetries of RSK

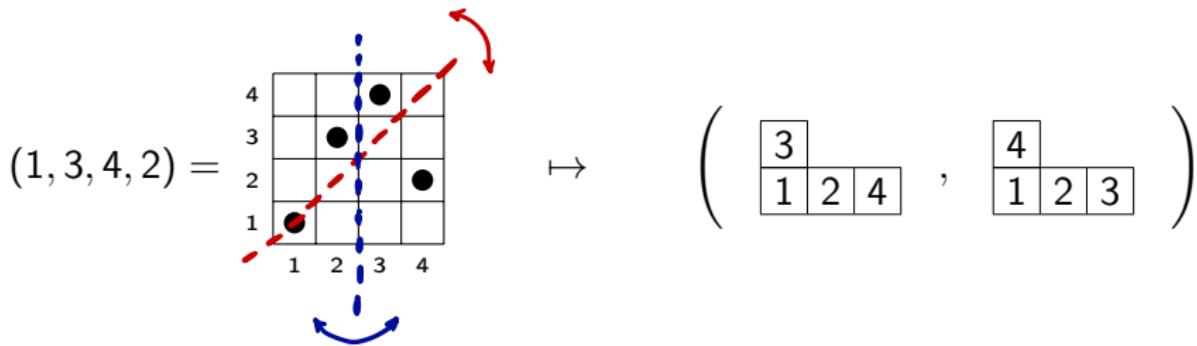


exhibit A
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repr. → random diagrams
○

shape ↔ character
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exhibit B
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RSK
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bumping routes
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the end
○○

magic symmetries of RSK

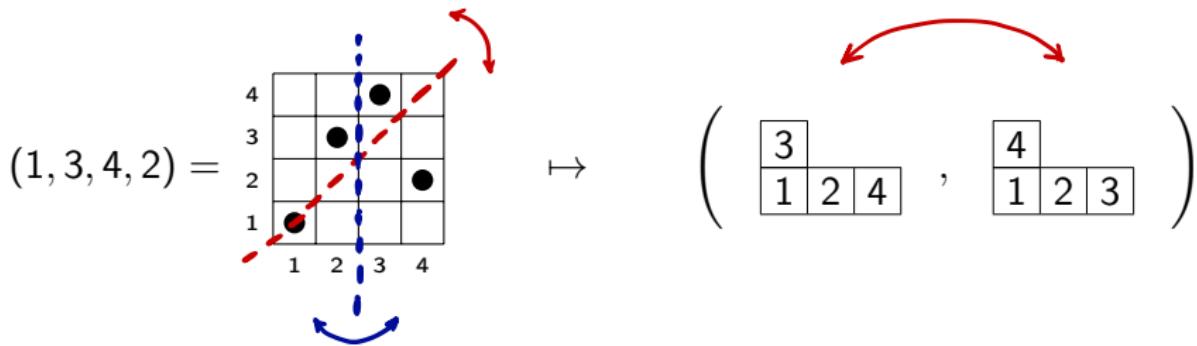


exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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RSK
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bumping routes
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the end
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magic symmetries of RSK

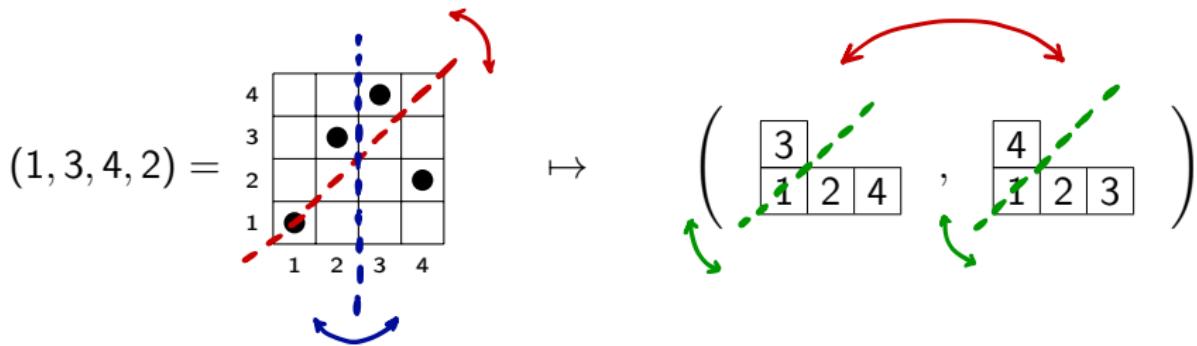


exhibit A
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shape ↔ character
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exhibit B
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bumping routes
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magic symmetries of RSK

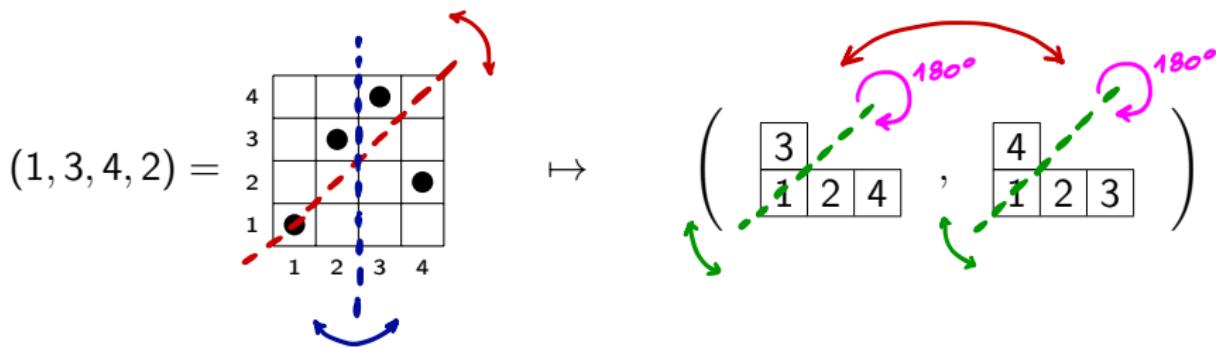


exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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RSK
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bumping routes
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the end
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exhibit B: how it was created?

let w_1, \dots, w_n be independent random variables
with the uniform distribution on $[0, 1]$

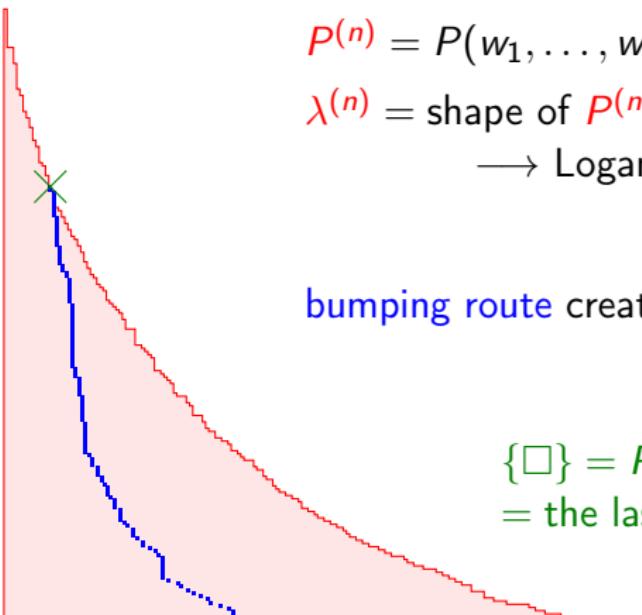
$$0 \leq s \leq 1$$

$$P^{(n)} = P(w_1, \dots, w_n)$$

$$\lambda^{(n)} = \text{shape of } P^{(n)}$$

→ Logan, Shepp, Vershik, Kerov (1977)

bumping route created in the insertion $P^{(n)} \leftarrow s$
(in this example $s = 0.2$)


$$\{\square\} = P(w_1, \dots, w_n, s) \setminus P(w_1, \dots, w_n)$$

= the last box in the bumping route

exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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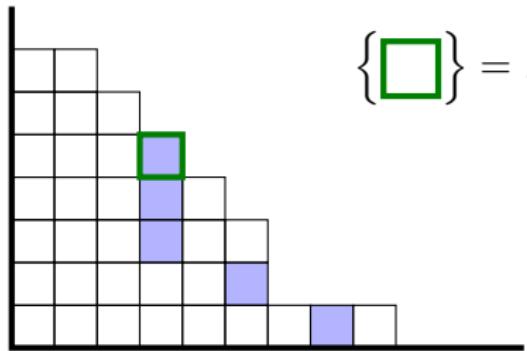
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bumping routes
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the end
○○

the end of the bumping route



$$\{\square\} = P(w_1, \dots, w_n, s) \setminus P(w_1, \dots, w_n)$$

Theorem (Dan Romik, Piotr Śniady
2015)

$$\frac{\square}{\sqrt{n}} \xrightarrow[n \rightarrow \infty]{\text{in probability}} (\text{RSKcos } s, \text{RSKsin } s)$$

exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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the end of the bumping route

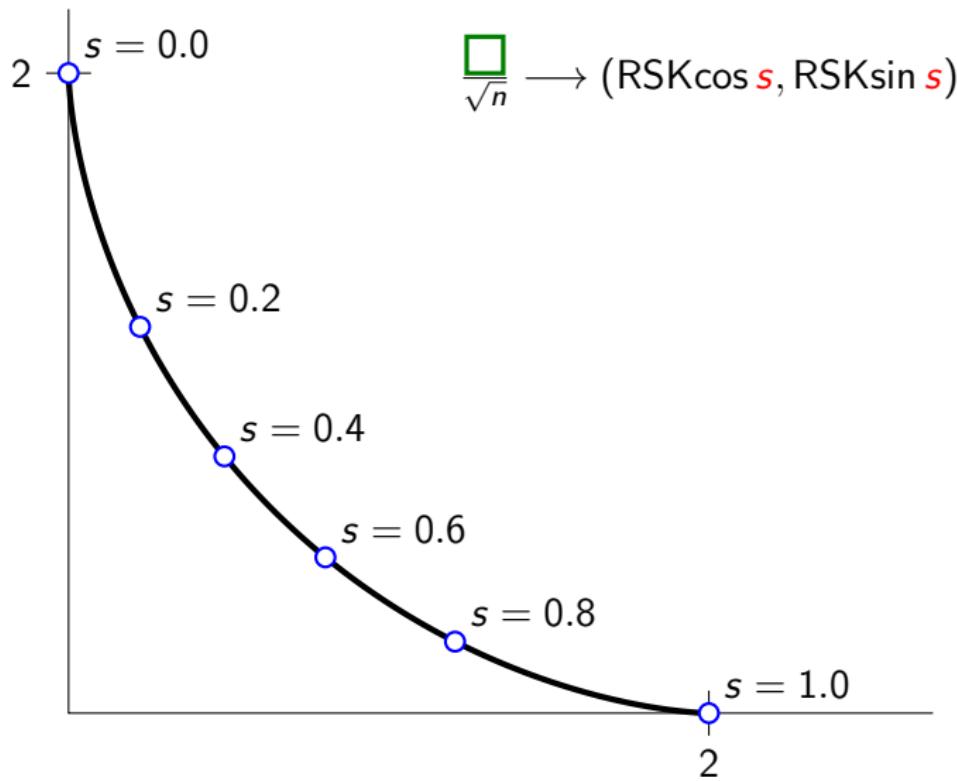


exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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the end
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the end of the bumping route

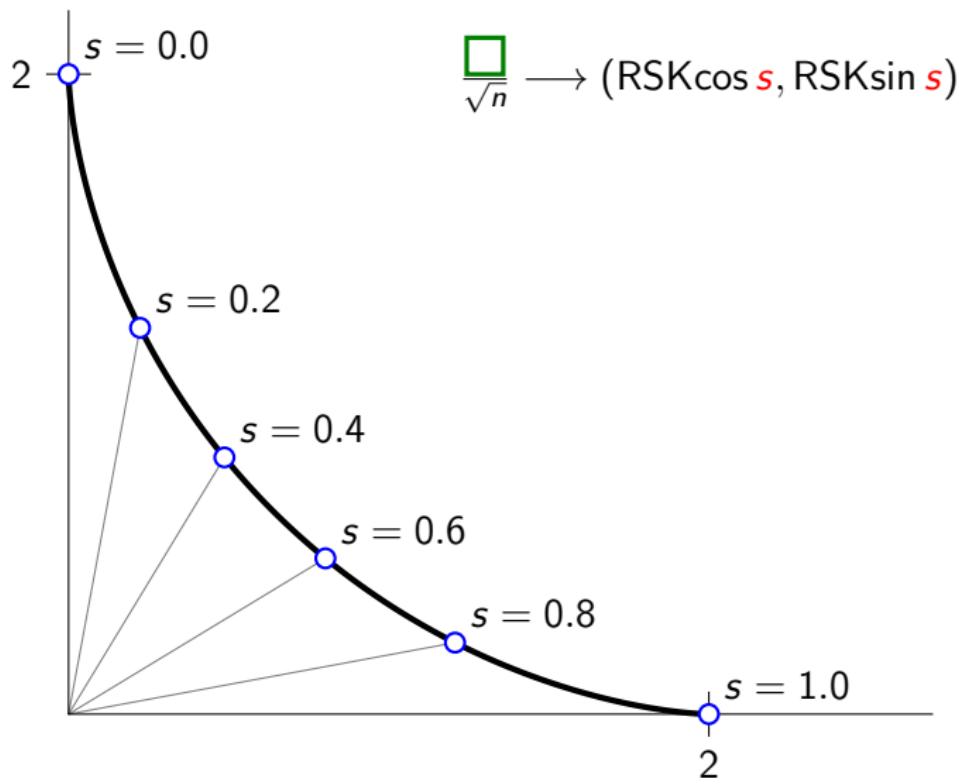


exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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the end
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the bumping route

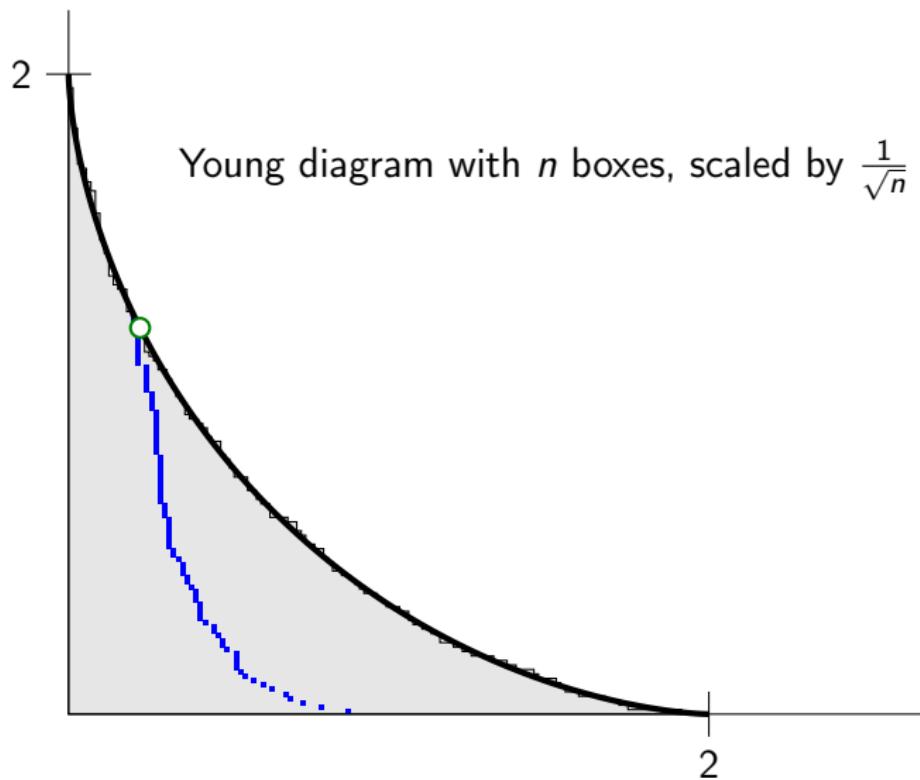


exhibit A
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repr. → random diagrams
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shape ↔ character
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bumping routes
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the end
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the bumping route

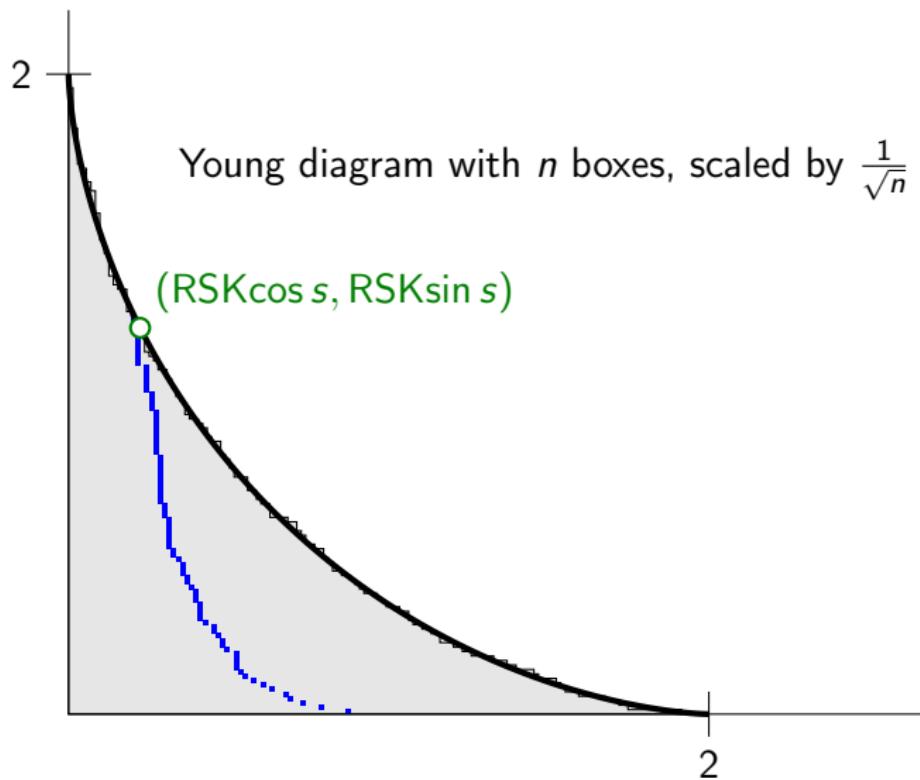


exhibit A
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repr. → random diagrams
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shape ↔ character
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bumping routes
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the end
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the bumping route

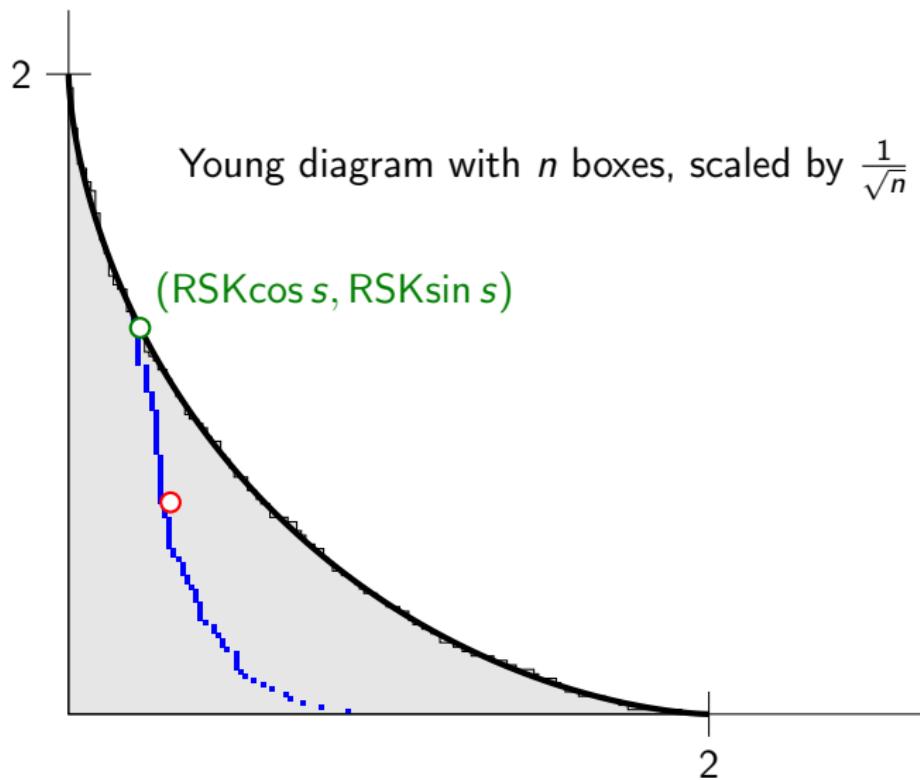


exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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the end
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the bumping route

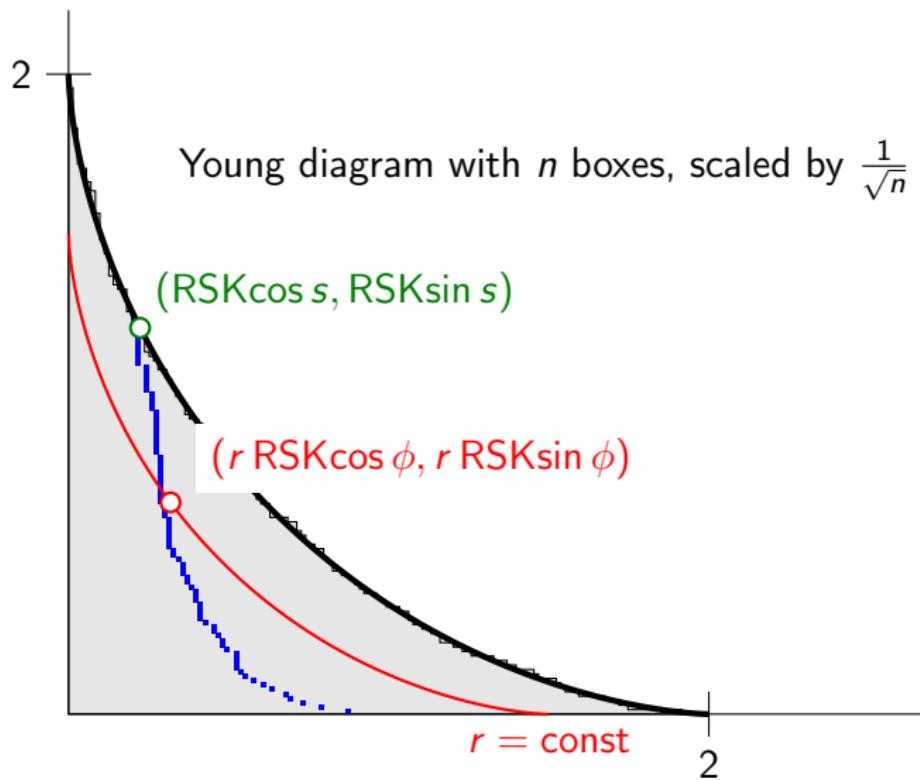


exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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the bumping route

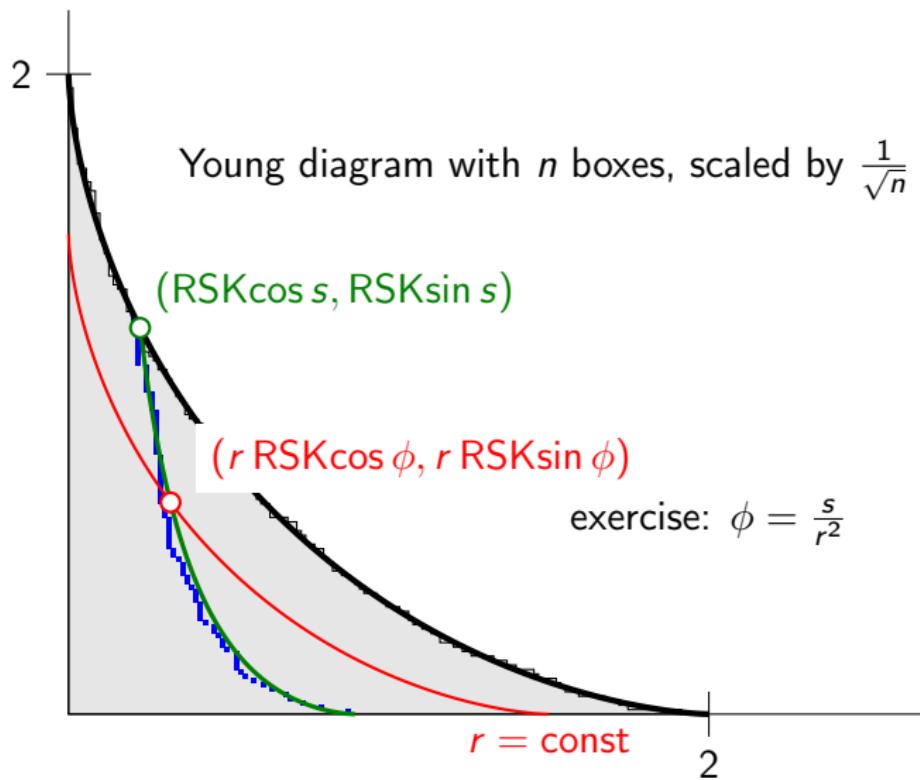


exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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the end
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diffusion of a box in the insertion tableau $P(w)$

→ Mikołaj Marciniak (2022)

exhibit A
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repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

exhibit B

RSK
0000

bumping routes

5

the end
oo

diffusion of a box in the insertion tableau $P(w)$

will this box ever reach the first column?

→ Marciak, Maślanka, Śniady 2021

exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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RSK
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bumping routes
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S_∞
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the end
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hydrodynamics of the insertion tableau $P(w)$

exhibit A
ooooorepr. → random diagrams
○shape ↔ character
ooooooooooooexhibit B
○RSK
oooobumping routes
ooooo S_∞
● oooo
the end
oorepresentation theory of S_n

- representation:

$$\rho: S_n \rightarrow \text{End}(V)$$

V is finite dimensional

- irreducible representations,
- irreducible characters,

repres. theory of $S_\infty = \bigcup_{n \geq 1} S_n$

- representation:

$$\rho: S_\infty \rightarrow B(\mathcal{H})$$

\mathcal{H} is a *Hilbert space*

- *factorial representations*
→ operator algebras
- *extremal characters*,

Vershik, Kerov:
link between

- factorial representations of S_∞ ,
- RSK applied to random input,
- **random infinite tableaux**,

exhibit A
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repr. → random diagrams
○

shape ↔ character
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exhibit B
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bumping routes
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S_∞
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the end
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infinite version of RSK

exhibit A
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repr.→random diagrams
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shape↔character
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exhibit B
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RSK
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bumping routes
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S_∞
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the end
○○

8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

jeu de taquin

start with (infinite) tableau

$t = Q(w_1, w_2, \dots)$,

exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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S_∞
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the end
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8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,

exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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the end
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8	13	18	32
6	9	12	23
4	5	7	19
2	3	10	

jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,

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repr. → random diagrams
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shape ↔ character
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bumping routes
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the end
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8	13	18	32
6	9	12	23
4	5	7	19
2	3	10	

jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,

exhibit A
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repr. → random diagrams
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shape ↔ character
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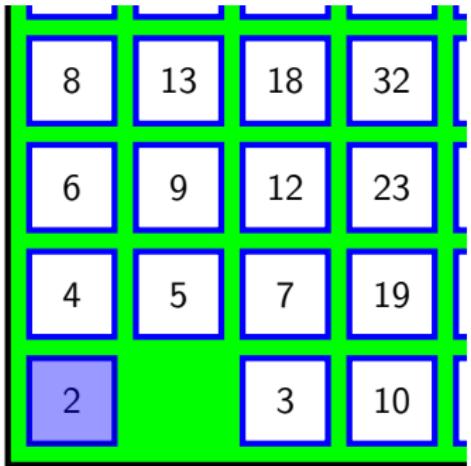
exhibit B
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bumping routes
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the end
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jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,

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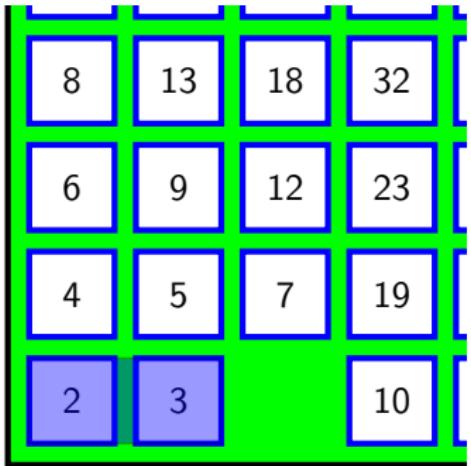
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bumping routes
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jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,

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repr.→random diagrams
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shape↔character
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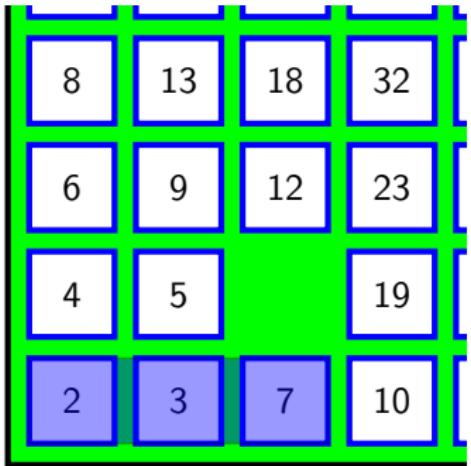
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bumping routes
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the end
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jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,

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repr. → random diagrams
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shape ↔ character
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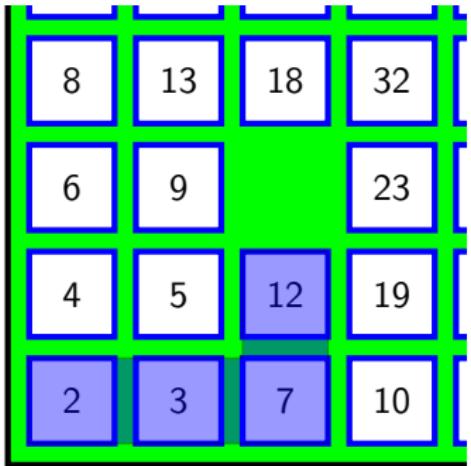
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bumping routes
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the end
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jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,

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repr. → random diagrams
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shape ↔ character
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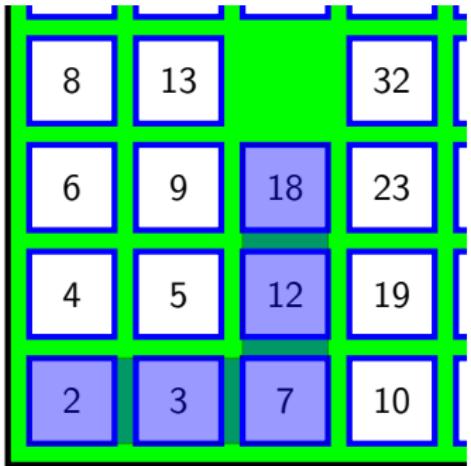
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bumping routes
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the end
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jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,

exhibit A
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repr. → random diagrams
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shape ↔ character
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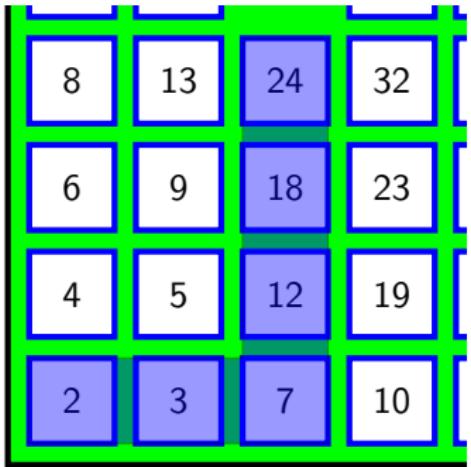
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bumping routes
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the end
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jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,

exhibit A
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repr. → random diagrams
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shape ↔ character
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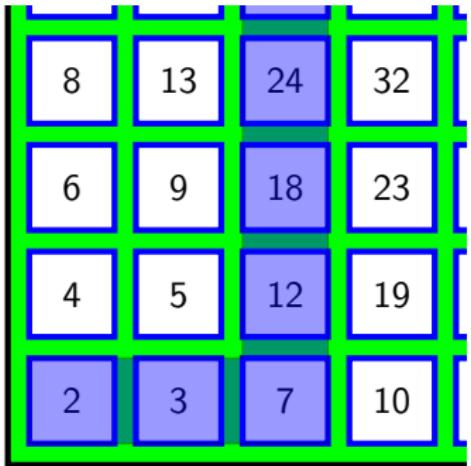
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bumping routes
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S_∞
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the end
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jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,

exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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S_∞
○○●○○

the end
○○

8	13	24	32
6	9	18	23
4	5	12	19
2	3	7	10

jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,
- ③ subtract 1 from all boxes

exhibit A
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repr. → random diagrams
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shape ↔ character
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exhibit B
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bumping routes
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S_∞
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the end
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7	12	23	31
5	8	17	22
3	4	11	18
1	2	6	9

jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,
- ③ subtract 1 from all boxes

exhibit A
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repr. → random diagrams
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shape ↔ character
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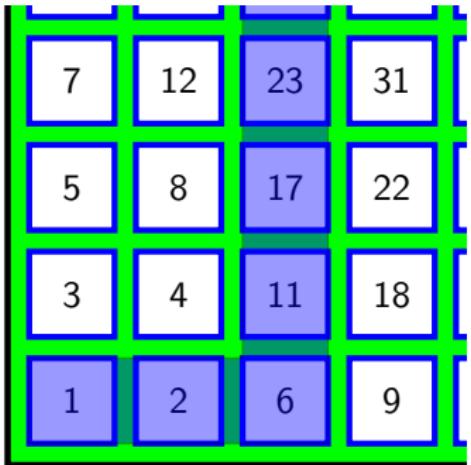
exhibit B
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bumping routes
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S_∞
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the end
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jeu de taquin

start with (infinite) tableau

$$t = Q(w_1, w_2, \dots),$$

- ① remove corner box,
- ② sliding,
- ③ subtract 1 from all boxes

output:

- new tableau = $Q(\text{WT}, w_2, w_3, \dots)$,
- blue trajectory

exhibit A
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repr. → random diagrams
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shape ↔ character
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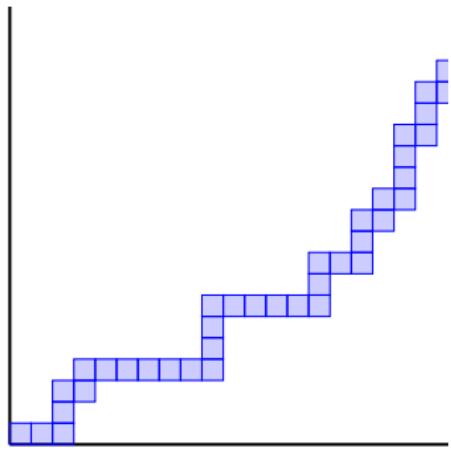
exhibit B
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bumping routes
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S_∞
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the end
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trajectory of jeu de taquin has an asymptote



if $t = Q(w_1, w_2, \dots)$ is a random infinite tableau ...

exhibit A

repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

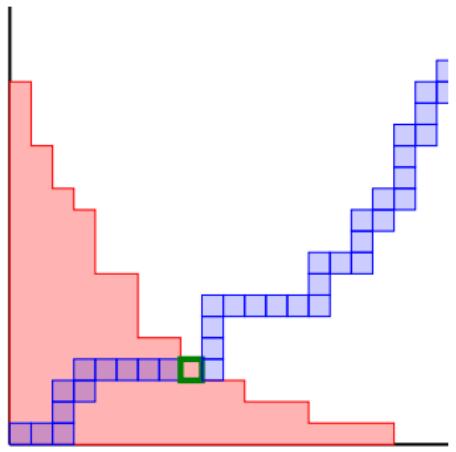
exhibit B

RSK
0000

bumping routes
oooooo

S_∞ the end
○○○●○○

trajectory of jeu de taquin has an asymptote



if $t = Q(w_1, w_2, \dots)$ is a random infinite tableau ...

$$\lambda^{(n)} = \{\text{boxes} \leq n\}$$

{  }

exhibit A

repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

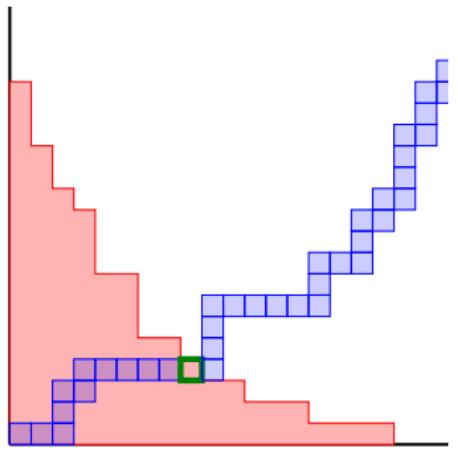
exhibit B

RSK
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bumping routes
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the end

trajectory of jeu de taquin has an asymptote



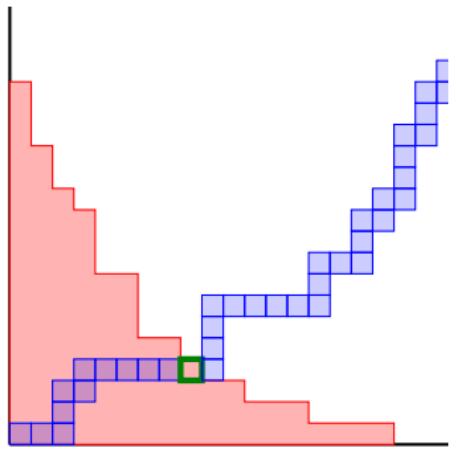
if $t = Q(w_1, w_2, \dots)$ is a random infinite tableau ...

$$\lambda^{(n)} = \{\text{boxes} \leq n\}$$

$$\{\square\} = Q(\textcolor{red}{w_1}, w_2, \dots, w_n) \setminus Q(w_2, \dots, w_n) =$$

exhibit A
○○○○○repr. → random diagrams
○shape ↔ character
○○○○○○○○○○○○exhibit B
○RSK
○○○○bumping routes
○○○○○ S_∞
○○○●○
the end
○○

trajectory of jeu de taquin has an asymptote

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$$\left\{ \square \right\} = Q(w_1, w_2, \dots, w_n) \setminus Q(w_2, \dots, w_n) =$$

$$Q(1 - w_n, \dots, 1 - w_1) \setminus Q(1 - w_n, \dots, 1 - w_2)$$

exhibit A

repr. → random diagrams
○

shape \leftrightarrow character
oooooooooooo

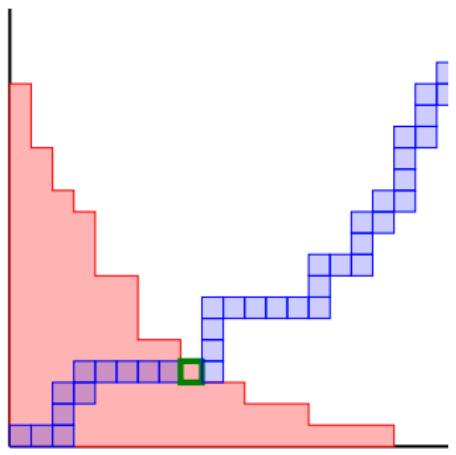
exhibit B

RSK
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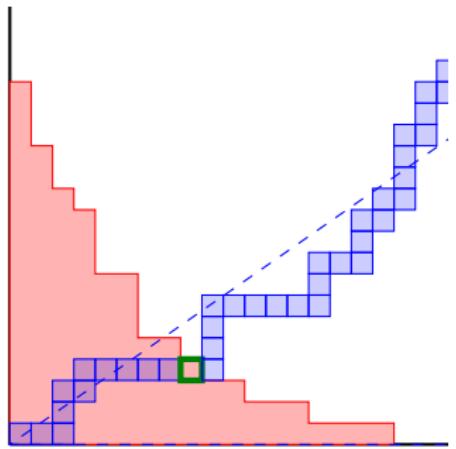
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$$Q(1-w_n, \dots, \color{red}{1-w_1}) \setminus Q(1-w_n, \dots, 1-w_2)$$

$$\approx \sqrt{n}(\text{RSKcos}(1 - w_1), \text{RSKsin}(1 - w_1))$$

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exhibit B
○

RSK
○○○○

bumping routes
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S_∞
○○○○●
the end
○○

jeu de taquin in action

→ Łukasz Maślanka, Piotr Śniady 2022

exhibit A
ooooo

repr. → random diagrams
○

shape ↔ character
oooooooooooo

exhibit B
○

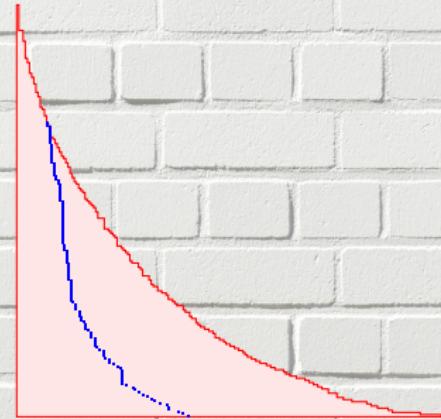
RSK
oooo

bumping routes
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S_∞
ooooo

the end
●○

conclusions



asymptotic / visual viewpoint may give new questions,
interesting from the algebraic combinatorics viewpoint

exhibit A
○○○○○

repr. → random diagrams
○

shape ↔ character
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exhibit B
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RSK
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bumping routes
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S_∞
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the end
○●

some coauthors

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transparencies, references, homework available on
<http://psniady.impan.pl/fpsac>